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Does the Realistic Interpretation of a Mathematical Superposition of States Imply a Mixture?

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Abstract Contrary to conventional view, it is shown that, for an ensemble of either single-particle systems or multi-particle systems, the realistic interpretation of a mathematical superposition of states that mathematically describes the ensemble does not imply that the ensemble is a mixture. Therefore it cannot be argued, as is conventionally done, that the realistic interpretation is wrong on the basis that some predictions derived from the mixture are different from the corresponding predictions derived from the mathematical superposition of states.

Keywords Superposition of states · Realistic interpretation · Mixture

1 Introduction

One of the central conceptual problems in quantum theory is [1, 2] the physical meaning of a mathematical superposition of states $\sum_k a_k|\varphi_k\rangle$ that mathematically describes an ensemble of either single-particle systems or multi-particle systems. In particular, what is the *physical* state of each ensemble member *prior* to measurement? There are [1, 2] three interpretations of the mathematical superposition $\sum_k a_k|\varphi_k\rangle$. The first interpretation: before measurement, $\sum_k a_k|\varphi_k\rangle$ is the physical state of each member of the ensemble. The second interpretation: before measurement, each member of the ensemble does not have a physical state. The third interpretation, which is the realistic interpretation: before measurement,

$|\varphi_k\rangle$ is the *physical* state of $|a_k|^2 \times 100$ percent of the systems in the ensemble.

According to conventional view, see [3–5] for example, the realistic interpretation of a mathematical superposition of states $\sum_k a_k|\varphi_k\rangle$, which mathematically describes an ensemble of either single-particle systems [3] or multi-particle systems [4, 5], implies that the ensemble

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is a mixture. By a mixture, it is meant [3–5] that

$|\varphi_k\rangle$ *mathematically* describes $|a_k|^2 \times 100$ percent of the systems in the ensemble.

There are instances [3–6] where the expectation value that is derived from the mixture is different from the value derived from the mathematical superposition of states $\sum_k a_k|\varphi_k\rangle$. One example of such a difference is the position expectation value; other examples are given in [3–6]. Differences between the expectation values derived from the mixture and from the mathematical superposition of states $\sum_k a_k|\varphi_k\rangle$ have led to the conventional conclusion, see [5] for example, that the realistic interpretation of a mathematical superposition of states is incorrect.

The aim of this paper is neither to argue nor prove that the realistic interpretation of a mathematical superposition of states $\sum_k a_k|\varphi_k\rangle$ is correct, as some supporters of the interpretation have done previously (e.g., see the discussion in [7]). The aim of this paper is only to investigate whether the realistic interpretation of a mathematical superposition of states, which mathematically describes an ensemble of systems, really implies the ensemble is a mixture. I will show that, in both the single-particle case (Sect. 2) and multi-particle case (Sect. 3), the realistic interpretation of a mathematical superposition of states (i.e., $|\varphi_k\rangle$ is the physical state of $|a_k|^2 \times 100$ percent of the systems in the ensemble before measurement) does not imply a mixture (i.e., $|\varphi_k\rangle$ mathematically describes $|a_k|^2 \times 100$ percent of the systems in the ensemble), contrary to conventional view. An immediate corollary of this conclusion is also discussed at the end of the paper.

2 Single-Particle Case

We begin with the single-particle case. Consider, for simplicity, an ensemble of single-particle systems that is mathematically described by this mathematical superposition of states:

$$\psi = \sqrt{\frac{2}{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle. \tag{1}$$

According to the realistic interpretation of ψ, prior to measurement, $|0\rangle$ is the physical state of two-thirds of the ensemble and $|1\rangle$ is the physical state of the remainder one-third of the ensemble.

First, for simplicity, let's consider the case where there are only three single-particle systems, S1, S2 and S3, in the ensemble. The first possibility is that, prior to measurement, the physical state of S1 is $|0\rangle$, the physical state of S2 is $|0\rangle$, and the physical state of S3 is $|1\rangle$. The second possibility is that, prior to measurement, the physical state of S1 is $|0\rangle$, the physical state of S2 is $|1\rangle$, and the physical state of S3 is $|0\rangle$. The third and final possibility is that, prior to measurement, the physical state of S1 is $|1\rangle$, the physical state of S2 is $|0\rangle$, and the physical state of S3 is $|0\rangle$. So, although $|0\rangle$ is the physical state of two of the three systems in the ensemble and $|1\rangle$ is the physical state of one of the three systems in the ensemble prior to measurement, it is not known exactly which two systems have the physical state $|0\rangle$ and exactly which system has the physical state $|1\rangle$ prior to measurement. From the three possibilities, it follows that, prior to measurement, the physical state of S1 is either $|0\rangle$ with probability $2/3$ or $|1\rangle$ with probability $1/3$, the physical state of S2 is also either $|0\rangle$ with probability $2/3$ or $|1\rangle$ with probability $1/3$, and the physical state of S3 is also either $|0\rangle$ with probability $2/3$ or $|1\rangle$ with probability $1/3$. Thus, from a realistic viewpoint, each system (S1, S2, S3) in the ensemble is mathematically described by the mathematical superposition of states ψ in (1) and *not* mathematically described by a basis state $|0\rangle$ or $|1\rangle$, consistent with the fact that the ensemble is mathematically described by ψ.

The same conclusion holds for each system for an arbitrary number of systems in the ensemble. Therefore, the realistic interpretation of the mathematical superposition of states ψ($|0\rangle$ is the physical state of two-thirds of the ensemble and $|1\rangle$ is the physical state of the remainder one-third of the ensemble) does not imply that the ensemble is a mixture ($|0\rangle$ mathematically describes two-thirds of the ensemble and $|1\rangle$ mathematically describes the remainder one-third of the ensemble).

A similar argument for any other mathematical superposition of states (i.e., a superposition of any number of basis states with any coefficients) that mathematically describes an ensemble of single-particle systems leads to the same conclusion: the realistic interpretation of the mathematical superposition of states does not imply that the ensemble is a mixture.

3 Multi-Particle Case

The argument for the multi-particle case is similar to the one for the single-particle case and I will illustrate it for an ensemble of two-particle systems that is mathematically described by this mathematical superposition of states:

$$\phi = \frac{1}{\sqrt{2}}|0\rangle|0\rangle + \frac{1}{\sqrt{2}}|1\rangle|1\rangle. \tag{2}$$

According to the realistic interpretation of ϕ, prior to measurement, $|0\rangle|0\rangle$ is the physical state of half of the ensemble and $|1\rangle|1\rangle$ is the physical state of the other half of the ensemble.

For simplicity, let's first consider an ensemble of only two two-particle systems, T1 and T2. In this case, there are only two possibilities: prior to measurement, $|0\rangle|0\rangle$ is the physical state of T1 (i.e., $|0\rangle$ is the physical state of each of the two particles of system T1) and $|1\rangle|1\rangle$ is the physical state of T2 (i.e., $|1\rangle$ is the physical state of each of the two particles of system T2), *or*, $|1\rangle|1\rangle$ is the physical state of T1 and $|0\rangle|0\rangle$ is the physical state of T2. So, although $|0\rangle|0\rangle$ is the physical state of one of the two systems in the ensemble and $|1\rangle|1\rangle$ is the physical state of the other system in the ensemble prior to measurement from a realistic viewpoint, it is not known exactly which system has the physical state $|0\rangle|0\rangle$ and exactly which system has the physical state $|1\rangle|1\rangle$ prior to measurement. The two possibilities imply that, prior to measurement, the physical state of T1 is either $|0\rangle|0\rangle$ or $|1\rangle|1\rangle$ with equal probability of $1/2$, and the physical state of T2 is also either $|0\rangle|0\rangle$ or $|1\rangle|1\rangle$ with equal probability of $1/2$. So, from a realistic viewpoint, T1 and T2 are each mathematically described by the mathematical superposition of states ϕ in (2) and not by a basis state $|0\rangle|0\rangle$ or $|1\rangle|1\rangle$.

Likewise, for an ensemble of an arbitrary number of systems, each member of the ensemble is, from a realistic viewpoint, mathematically described by the mathematical superposition of states ϕ and not mathematically described by a basis state. Therefore, the realistic interpretation of the mathematical superposition of states ϕ ($|0\rangle|0\rangle$ is the physical state of half of the ensemble and $|1\rangle|1\rangle$ is the physical state of the other half of the ensemble) does not imply that the ensemble is a mixture ($|0\rangle|0\rangle$ mathematically describes half of the ensemble and $|1\rangle|1\rangle$ mathematically describes the other half of the ensemble).

4 Conclusions and Ending Remarks

In summary, I have shown that, for an ensemble of either single-particle systems or multi-particle systems, the realistic interpretation of a mathematical superposition of states that mathematically describes the ensemble does not imply that the ensemble is a mixture, contrary to conventional view.

The conclusion above has an immediate corollary. Because the realistic interpretation of a mathematical superposition of states that mathematically describes an ensemble does not imply that the ensemble is a mixture, it cannot be argued, as is conventionally done, that the realistic interpretation is wrong on the basis that some predictions (expectation values) derived from the mixture are different from the corresponding predictions derived from the mathematical superposition of states. Whether the realistic interpretation is correct is therefore still an open question.

Inspired by a wave of recently proposed local realistic theories [8–12] for the Einstein-Podolsky-Rosen-Bohm (EPRB) experiment, I have shown how the realistic interpretation could be utilized to construct a realistic theory that is both local and consistent with the quantum mechanical predictions for the EPRB experiment [13] and also the Greenberger-Horne-Zeilinger (GHZ) experiment [14]. Other utilities of the realistic interpretation are being explored.

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